MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

PEM0036 – CALCULUS

(Foundation in Engineering)

13 MARCH 2019
9:00 a.m. – 11:00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 6 pages.

2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided. All necessary working **MUST** be shown.

4. Only **NON-PROGRAMMABLE** calculator is allowed.

APPENDIX

BASIC DIFFERENTIATION AND INTEGRATION FORMULAS

$$\frac{d}{dx}[\sin x]=\cos x \qquad \frac{d}{dx}[\cos x]=-\sin x$$

$$\frac{d}{dx}[\tan x]=\sec^2 x \qquad \frac{d}{dx}[\cot x]=-\csc^2 x$$

$$\frac{d}{dx}[\sec x]=\sec x\tan x \qquad \frac{d}{dx}[\csc x]=-\csc x\cot x$$

$$\frac{d}{dx}[e^x]=e^x \qquad \frac{d}{dx}[\ln x]=\frac{1}{x};\quad x>1$$

$$\frac{d}{dx}[\sin^{-1}x]=\frac{1}{\sqrt{1-x^2}};\quad |x|<1 \qquad \frac{d}{dx}[\cos^{-1}x]=-\frac{1}{\sqrt{1-x^2}};\quad |x|<1$$

$$\frac{d}{dx}[\tan^{-1}x]=\frac{1}{1+x^2} \qquad \frac{d}{dx}[\cot^{-1}x]=-\frac{1}{1+x^2}$$

$$\frac{d}{dx}[\sec^{-1}x]=\frac{1}{|x|\sqrt{x^2-1}};\quad |x|>1 \qquad \frac{d}{dx}[\csc^{-1}x]=-\frac{1}{|x|\sqrt{x^2-1}};\quad |x|>1$$

$$Area=\int_a^b [f(x)-g(x)]\ dx$$

$$Volume\ (Disk)=\pi\int_a^b [f(x)]^2\ dx$$

$$Volume\ (Washer)=\pi\int_a^b [f(x)]^2-[g(x)]^2\ dx$$

$$Volume\ (Cylindrical\ Shells)=\int_a^b 2\pi(shell\ radius)(shell\ height)\ dx$$

Continued...

QUESTION 1 [25 marks]

a. Evaluate the following limits:

i. $\lim_{x\to\infty}\left(\frac{5x}{x-1}+\frac{4x^2-3}{x^3-1}\right)$ [3 marks]

ii. $\lim_{x\to 0}\left(\frac{\sqrt[4]{x+16}-2}{x}\right)$ $\left[\text{Hint : Let } x+16=h^4 \ , \ h>0\right]$ [5 marks]

b. For the following functions, determine the respective limit using L'Hopital's Rule:

i. $\lim_{x\to 5}\left(\frac{5x-x^2}{x^2-2x-15}\right)$ [2 marks]

ii. $\lim_{x\to 0}\left(\frac{x-\sin(x)}{x^2}\right)$ [2 marks]

c. Consider the function $f(x)=\begin{cases}9-x^2 & \text{if} \quad x<2\\ \\ 1+x^2 & \text{if} \quad x>2\end{cases}$.

i. Find $\lim_{x\to 2} f(x)$. [3 marks]

ii. Is $f(x)$ continuous at $x = 2$? Explain why. [2 marks]

d. Check whether the following functions have horizontal/vertical/slant asymptote.

i. $f(x)=\frac{\frac{1}{x}-\frac{1}{3}}{x-3}$ [4 marks]

ii. $f(x)=\frac{x^3+1}{x^2-1}$ [4 marks]

Continued...

QUESTION 2 [25 marks]

a. Find $\frac{dy}{dx}$ of

i. $y = 4x(x+1)$. [2 marks]

ii. $\ln(y) = e^x + xy$ at $(0, e)$. [4 marks]

b. Let $x = 2t^2 - t^3$ be the position of a particle in meters after t seconds.

i. What does $\frac{dx}{dt}$ represent? [1 mark]

ii. What is the velocity at $t = 1$? [2 marks]

iii. What is the acceleration at $t = 0$? [2 marks]

iv. What is the velocity when acceleration is zero? [3 marks]

c. The tangent to the curve $y = 3x^2$ at point P has gradient 12.

i. Find the point P. [2 marks]

ii. Find the equation of the normal at P. [2 marks]

d. Let function $f(x) = x + \frac{4}{x}$. Find the

i. critical number(s) of $f(x)$. [4 marks]

ii. interval(s) on which $f(x)$ is increasing and /or decreasing. [3 marks]

Continued...

QUESTION 3 [25 marks]

a. Integrate the following:

i. $\int \left(2x+5e^{5x}-\frac{1}{\sqrt{x}}+\cos x\right) dx.$ [2 marks]

ii. $\int_5^6 (x+1)(x-4)^{20}\, dx.$ [5 marks]

b. Use integration by parts to evaluate $\int (2x^2)(\ln x^2)\, dx.$ [5 marks]

c. A region $\boldsymbol{R}$ (Figure Q3c) is enclosed by the curve $y=-x^2+9$ and the line $y=x+3$.

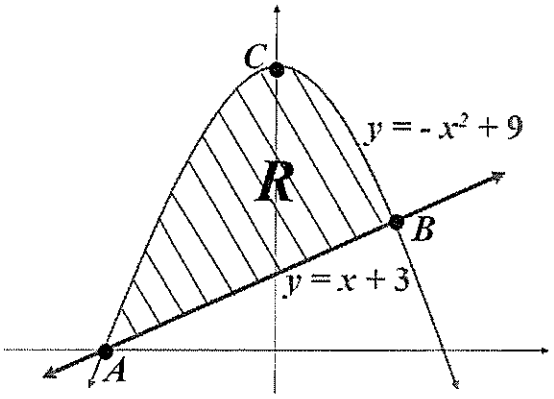

Figure Q3c

i. Show that the points $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are (-3, 0), (2, 5) and (0, 9) respectively. [4 marks]

ii. Find the area of the shaded region $\boldsymbol{R}$. [4 marks]

iii. Use cylindrical shells method to find the volume of the resulting solid when the shaded region $\boldsymbol{R}$ is revolved about the line $x = 2$. [5 marks]

Continued...

QUESTION 4 [25 marks]

a. Find the function that satisfies the following initial value problem.

$$(\csc x)\left(\frac{dy}{dx}\right)=\frac{2e^{2x}}{\sin x} \quad ; \quad y(0)=3. \qquad \text{[5 marks]}$$

b. Solve the first order differential equation:

$$\left(\sqrt{\frac{x^2}{4}}\right)\left(\frac{dy}{dx}\right)=4x-y. \qquad \text{[8 marks]}$$

c. Solve the following differential equation:

$$y''-6y'+10y=10-e^{x}. \qquad \text{[12 marks]}$$

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